Form Factors and Long-Distance Effects in $B \to V(P)\ell^+\ell^-$ and $B \to V\gamma$

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I overview the hadronic input for the exclusive flavour-changing neutral-current B-decays with a vector ($V = K^*, \rho$) or pseudoscalar ($P = K, \pi$) meson in the final state. After presenting the current status of $B \to P, V$ form factors, I discuss the estimate of the charm-loop effect in $B \to K^{(*)}\ell^+\ell^-$ and $B \to K^*\gamma$.

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1 Introduction

The exclusive $B \to V(P)\,\ell^+\ell^-$ and $B \to V\gamma$ decays, with a vector $(V = K^*, \rho, ...)$ or pseudoscalar $(P = K, \pi, ...)$ meson in the final state are important for the search for flavour-changing new physics. I will overview the current status of the hadronic input for these decays. In Standard Model (SM), the CKM favoured $B \to K^{(*)}\ell^+\ell^-$ decay amplitudes reduce to the matrix elements

$$A(B \to K^{(*)}\ell^+\ell^-) = \langle K^{(*)}\ell^+\ell^- \mid H_{eff} \mid B \rangle \tag{1}$$

of the effective Hamiltonian

$$H_{eff} = -\frac{4G_F}{\sqrt{2}}V_{tb}V_{ts}^*\sum_{i=1}^{10}C_i(\mu)O_i(\mu)\,, \tag{2}$$

where the small $\sim V_{ub}V_{us}^*$ part is neglected, and the dominant $b \to s$ operators are $O_{9,10}$ and O_7, with the Wilson coefficients $C_9(m_b) \simeq 4.2$, $C_{10}(m_b) \simeq -4.4$ and $C_7(m_b) \simeq -0.3$, respectively (for a review see, e.g., [1]).

The hadronic matrix elements of these operators factorize, e.g., the contribution of the operator $O_9 = (\overline{s}_L\gamma_\rho b_L)(\overline{\ell}\gamma^\rho\ell)$ to the decay amplitude (1) reduces to the hadronic matrix elements $\langle K^{(*)}(p)|\overline{s}_L\gamma_\rho b_L|B(p+q)\rangle$, parameterized in terms of the $B \to K^{(*)}$ form factors depending on q^2, the momentum transfer to the lepton pair. For the $B \to V\gamma$ decay one correspondingly needs the form factors at $q^2 = 0$.

In order to access the flavour-changing neutral current (FCNC) interaction encoded in (2) and to trace and/or constrain new physics, one has to compare the measured exclusive decay observables with the SM predictions. For the latter, an accurate knowledge of the $B \to P, V$ form factors is necessary but not sufficient. Important are also the contributions to the $B \to V(P)\ell^+\ell^-$ and $B \to V\gamma$ decay amplitudes due to the operators $O_{1,2,...,6,8g}$, which have to be analyzed one by one (see e.g. [2]) and added to the dominant FCNC contributions. Especially important are the current-current operators $O_1^{(c)} = (\overline{s}_L\gamma_\rho c_L)(\overline{c}_L\gamma^\rho b_L)$ and $O_2^{(c)} = (\overline{s}_L^j\gamma_\rho c_L^i)(\overline{c}_L^i\gamma^\rho b_L^j)$ with large Wilson coefficients $C_1(m_b) \simeq 1.1$, $C_2 \simeq -0.25$. Combined with the e.m. interactions of quarks and leptons, they generate $b \to s$ transitions with intermediate c-quark loops. The resulting hadronic matrix elements contain nonfactorizable parts, not reducible to $B \to P, V$ form factors. In the following Sect. 2, I will discuss the form factors and in Sect. 3 the results of the recent analysis [3] of the charm-loop effect in $B \to K^{(*)}\ell^+\ell^-$ and $B \to K^*\gamma$.

2 $B \to P, V$ form factors

The form factors of $\overline{s}_L\gamma_\mu b_L$ and $\overline{d}_L\gamma_\mu b_L$ currents involved in $B \to P(V)\,\ell^+\ell^-$ decays are related, via $SU(3)_{fl}$ and isospin symmetry, respectively, to the form factors

of $\overline{u}_L\gamma_\mu b_L$ current. The latter can in principle be determined from the measured $B \to \pi(\rho)\ell\nu_\ell$ semileptonic widths, with $|V_{ub}|$ taken from the inclusive semileptonic measurements. However, for the $B \to K^{(*)}$ form factors this way of determination cannot be sufficiently accurate, because $SU(3)_{fl}$ is violated up to $\sim 20\%$ (like in the ratios f_K/f_π, $f_{DK}(0)/f_{D\pi}(0)$). The heavy-quark limit ($m_b, m_c \to \infty$) provides another useful flavour symmetry, predicting nontrivial relations between $B-$ and $D-$ meson hadronic amplitudes. Hence, in principle, one can try to obtain the $B \to P, V$ form factors employing the $D \to P, V$ form factors extracted from the exclusive semileptonic D decays. Again, to achieve a reasonable accuracy, one has to assess the symmetry violating, $\sim 1/m_{c,b}$ corrections. They are generally not small. E.g., QCD calculations of the B and D decay constants yield $f_B \sim f_D \sim 200$ MeV, whereas in the heavy-quark limit $f_D/f_B \sim \sqrt{m_b/m_c} \sim \sqrt{3}$. We come to a conclusion that in order to reach $< 20\%$ accuracy one needs a direct calculation of the $B \to P, V$ form factors in full QCD.

Currently, lattice QCD with 3 dynamical flavours has achieved a $\sim 10\%$ accuracy, but only for the $B \to \pi$ form factors in the region of large $q^2 > 15$ GeV^2. The future goal is to reach 5% accuracy (see e.g.,[4]). The $B \to K$ form factors have been obtained recently [5] in the quenched approximation. The lattice calculations of $B \to V$ form factors ($V = \rho, \omega, K^*$) are complicated due to instability of vector mesons, and only earlier results in quenched approximation are available.

In the important region of small and intermediate q^2 (large and moderate recoil of the final meson) the $B \to P, V$ form factors are calculated from QCD light-cone sum rules (LCSR). This technique is used for finite quark masses and, therefore takes into account the flavour-symmetry violation. The key nonperturbative objects are the light-cone distribution amplitudes (DAs) of the light P- or V-meson. The LCSR method and results for $B \to \pi$ form factors are overviewed in [6]. The most recent calculation for the $B \to \pi$ form factor is in [7], the $B \to K$ form factors were updated in [8]. A typical uncertainty is about 15%, with a little room for improvement. The same method and input successfully reproduce the $D \to \pi$ and $D \to K$ form factors, as shown in [9]. The LCSR results for $B_{(s)} \to V$ form factors ($V = \rho, \omega, K^*, \phi$) are available from [10]. Note that the instability of the vector meson V (the $\rho \to \pi\pi$ or $K^* \to K\pi$ widths) are neglected also in the LCSR calculation.

Alternative LCSR's for $B \to P, V$ form factors are obtained with B-meson distribution amplitudes [11] taken as a nonperturbative input. Here all pseudoscalar and vector mesons are treated on equal footing, being interpolated by a corresponding light-quark current. The overall accuracy of these sum rules is somewhat less than of the conventional LCSR's with DA's of light mesons. The gluon radiative corrections are not yet calculated and the uncertainties of the parameters of B-meson DA's are still large.

Among the non-lattice tools for $B \to P, V$ form factors are also effective theories (HQET, QCD factorization, SCET) where non-trivial relations between the form

factors in the large recoil limit are predicted. A comprehensive analysis of $B \to K^* \ell^+ \ell^-$ in terms of this approach can be found, e.g., in [2], where the contributions with hard gluons to the decay amplitudes are identified and calculated. The soft $B \to P, V$ form factors defined in the heavy-quark and large-recoil limit and the B, P, V-meson DA's represent the external input. LCSR in SCET [12] can be used to calculate the soft form factors. Further increasing the accuracy in the effective theories demands taking into account the power-suppressed contributions.

In addition to the calculational methods, the analytical properties of the $B \to P, V$ form factors are employed, in the form of "series-parametrization". The idea is to map the complex q^2-plane onto the plane of the new variable $z(q^2)$, so that $|z| \ll 1$ in semileptonic region $0 < q^2 < (m_B - m_{P(V)})^2$. Hence, a Taylor expansion around $z = 0$ describes the form factor with a reasonable accuracy, allowing one to inter/extrapolate the calculated form factor beyond the region of validity of lattice QCD or LCSR. The latest version of this parameterization was introduced in [13] where one can find all details. A recent analysis of all form factors relevant for $B \to K^{(*)} \ell^+ \ell^-$, combining LCSR and available lattice QCD results with series parameterization can be found in [14].

Summarizing, the current uncertainty of $B \to P, V$ form factors is $12 - 15\%$, whereas $B \to V$ form factors have an additional "systematic error" related to the instability of vector mesons.

3 Charm loops in $B \to K^{(*)} \ell^+ \ell^-$
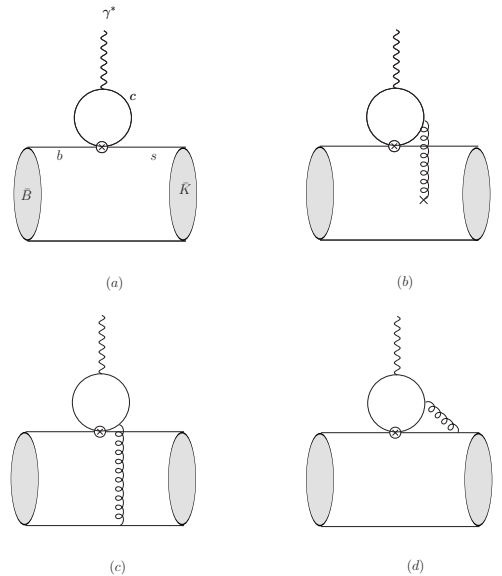

Figure 1: *Charm-loop effect in $B \to K^{(*)} \ell^+ \ell^-$: (a)-the leading-order factorizable contribution; (b) nonfactorizale soft-gluon emission, (c),(d)-hard gluon exchange.*

In addition to the FCNC contributions containing $B \to P, V$ form factors, the $B \to V(P) \ell^+ \ell^-$ and $B \to V \gamma$ decay amplitudes are "contaminated" by the effects of

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weak interaction combined with e.m. interaction. Let us discuss the most important "charm-loop" effect in $B \to K^{(*)}\ell^+\ell^-$ and $B \to K^*\gamma$, generated by the current-current operators $O_{1,2}^{(c)}$, acting together with the c-quark electromagnetic current (see Fig. 1). In $B \to K^{(*)}\ell^+\ell^-$, this mechanism involves an intermediate "charm-loop", coupled to the lepton pair via the virtual photon. In $B \to K^*\gamma$, the charm-loop is also possible if there is an additional gluon exchange with the rest of quarks.

The simple c-quark loop diagram (Fig. 1a) is usually included in the factorization formula for $B \to K^{(*)}\ell^+\ell^-$. In addition, hard-gluon exchanges between the c-quark loop and the rest of the diagram (Fig. 1c,d) are taken into account, together with other perturbative nonfactorizable effects (see e.g., [2]). One generally predicts these effects to be small, if q^2 is far below the charmonium region. The natural question is: how important are the contributions of the soft gluons emitted from the c-quark loop ? (Fig.1b) A related question concerns the validity of the approximation "c-quark-loop plus corrections" at large q^2, approaching the charmonium resonance region. Note that at $q^2 = m_\psi^2$, where $\psi = J/\psi, \psi(2S), ...$ is one of the vector charmonium states, the process $B \to K^{(*)}\ell^+\ell^-$ transforms into a nonleptonic weak decay $B \to \psi K^{(*)}$, followed by the leptonic annihilation of ψ. To avoid this "direct" charmonium background, the q^2-intervals around J/ψ and $\psi(2S)$ are subtracted from the measured lepton-pair mass distributions in $B \to K^{(*)}\ell^+\ell^-$. Nevertheless, the intermediate and/or virtual $\bar{c}c$ states contribute outside the resonance region and their effect has to be accurately estimated.

In [3] these two questions were addressed, employing the expansion near the light-cone of the product of the two operators: $O_{1,(2)}^{(c)}$ and c-quark e.m. current. As demonstrated in detail in [3], this operator-product expansion is valid at $q^2 \ll 4m_c^2$, provided $2m_c \gg \Lambda_{QCD}$. The leading-order term of this expansion is reduced to the simple $\bar{c}c$-loop, resulting in the well-known loop function $g(m_c^2, q^2)$ multiplying the local operator $\bar{s}_L \gamma^\rho b_L$. The nontrivial effect is related to the one-gluon term which yields [3] a convolution

$$\tilde{\mathcal{O}}_\mu(q) = \int d\omega \, I_{\mu\rho\alpha\beta}(q, m_c, \omega) \bar{s}_L \gamma^\rho \delta[\omega - \frac{(in_+\mathcal{D})}{2}]\tilde{G}_{\alpha\beta} b_L \qquad (3)$$

of a nonlocal quark-antiquark-gluon operator with the calculable coefficient function $I_{\mu\rho\alpha\beta}$. In the above, $n_+\mathcal{D}$ is the light-cone projection (defined so that $q \sim \frac{m_b}{2}n_+$ in the rest frame of B) of the covariant derivative acting on the gluon field-strength tensor and $\tilde{G}_{\alpha\beta} = \frac{1}{2}\epsilon_{\alpha\beta\sigma\tau}G^{\sigma\tau}$. The explicit expression for this coefficient function is presented in [3]. As explained there in more detail, two and more soft-gluon contributions are suppressed by additional powers of $1/(4m_c^2 - q^2)$ with respect to the leading one-gluon term. The operator in (3) results from an effective resummation of the tower of local operators. In the local limit, at $q^2 = 0$, we recover the local operator of the charm-loop with soft gluon, taken into account first in [15] for the $B \to X_s\gamma$ inclusive width and in [16] for the $B \to K^*\gamma$ amplitude. In the adopted approximation, the calculation of
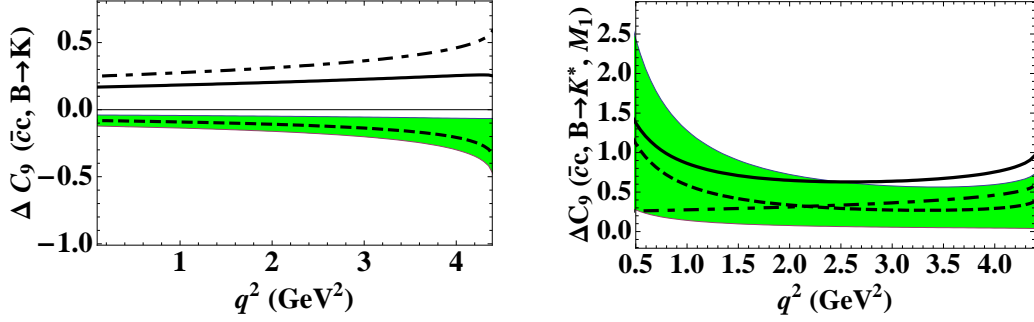
Figure 2: *The charm-loop effect in $B \to K\ell^+\ell^-$ (left panel) and $B \to K^*\ell^+\ell^-$ (right panel, one of the three amplitudes) expressed as a correction to the Wilson coefficient C_9 (solid), including the nonfactorizable soft-gluon contribution (dashed) with the shaded region indicating the estimated uncertainty and the factorizable contribution (dash-dotted).*

the charm-loop effect at small q^2 is then reduced to the two hadronic matrix elements. One of them is factorizable and expressed via $B \to K^{(*)}$ form factors. The soft-gluon emission contribution yields a hadronic matrix element of the nonlocal operator (3). This matrix element is calculated in [3] using the LCSR method [11] where the B-meson DA's (approximated in HQET) are used as a universal nonperturbative input.

The result for the charm-loop contribution to $A(B \to K\ell^+\ell^-)$ including the soft-gluon part is expressed in the form of a (process- and q^2-dependent) correction to the known Wilson coefficient C_9.

Figure 3: *The charm loop contribution to the Wilson coefficient C_9 for $\overline{B}_0 \to \overline{K}l^+l^-$ at q^2 below the open charm threshold, obtained from the dispersion relation fitted to the OPE result at $q^2 \ll 4m_c^2$. The central values are denoted by dashed line, shaded area indicates the estimated uncertainties.*
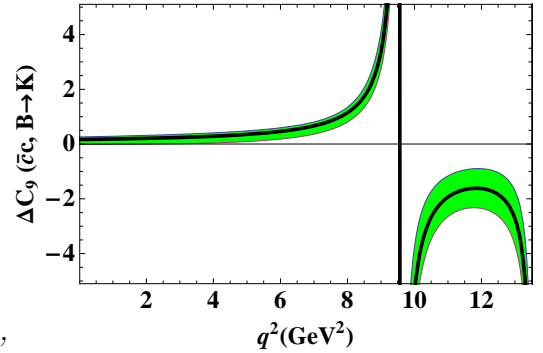
The calculated function $\Delta C_9^{(\bar{c}c,B \to K)}(q^2)$ plotted in Fig. 2[left] is valid at small $q^2 \ll 4m_c^2$. The numerical analysis reveals an important role of the soft-gluon part. It has an opposite sign with respect to the factorizable loop term. The result

$$\Delta C_9^{(\bar{c}c,B \to K)}(q^2 = 0, \mu \sim m_b) = 0.17^{+0.09}_{-0.18}, \tag{4}$$

has to be added to $C_9(\mu = m_b)$.

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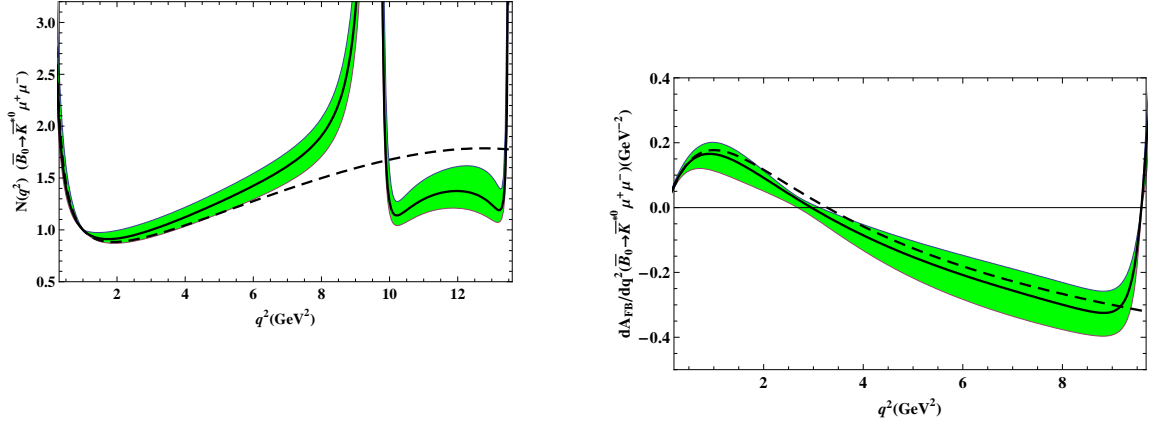Figure 4: *left:The differential width of $\overline{B}_0 \to \overline{K}^* \mu^+ \mu^-$ normalized at $q^2 = 1.0$ GeV^2, including the charm-loop effect calculated with the central values of input (solid, the shaded area indicates estimated uncertainties) and without this effect (dashed); right: The forward-backward asymmetry for $\overline{B}_0 \to \overline{K}^* \mu^+ \mu^-$ decay.*

For $B \to K^* \ell^+ \ell^-$ the effect is more pronounced and kinematically enhanced at small q^2 (see Fig. 2[right]). As a by-product of our calculation we also estimate the charm-loop effect in $B \to K^* \gamma$, where the factorizable loop vanishes and only the nonfactorizable gluon emission contributes.

Furthermore, to access large q^2 we use the dispersion relation in this variable for the invariant amplitudes determining the $B \to K^{(*)}$ hadronic matrix elements, saturating this relation with the first two charmonium levels. This relation is valid at any q^2, hence we can match it to the result of QCD calculation at $q^2 \ll 4m_c^2$. In addition, we fix the absolute values of the residues from experimental data on $B \to \psi K$ widths. The integral over the spectral density of higher states is then fitted as an effective pole. After fixing the parameters of the dispersion relation we predict the correction $\Delta C_9(q^2)$ at large q^2 (see Fig. 3).

Finally, the observables in $B \to K^{(*)} \ell^+ \ell^-$ are calculated employing the form factors and charm-loop amplitudes (see Fig.4). There is a moderate influence of the charm-loop effect on the position of the zero in the forward-backward asymmetry.

Concluding, I would like to emphasize that a careful analysis of all other similar effects (light-quark loops, weak annihilation etc.) including soft-gluon contributions

is necessary for obtaining a complete and accurate prediction for $B \to V(P)\ell^+\ell^-$ and $B \to V\gamma$ in SM.

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